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# Diffractive $S$ and $D$ -wave vector mesons in deep inelastic scattering

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## Abstract

We derive helicity amplitudes for diffractive leptonproduction of the  $S$  and  $D$  wave states of vector mesons. We predict a dramatically different spin dependence for production of the  $S$  and  $D$  wave vector mesons. We find very small  $R = \sigma_L/\sigma_T$  and abnormally large higher twist effects in production of longitudinally polarized  $D$ -wave vector mesons.

Diffractive vector meson production  $\gamma^* + p \rightarrow V + p'$ , in deep inelastic scattering (DIS) at small  $x = (Q^2 + m_V^2)/(W^2 + Q^2)$  is a testing ground of ideas on the QCD pomeron exchange and light-cone wave function (LCWF) of vector mesons ([1, 2, 3, 4, 5], for the recent review see [6]). (For the kinematics see fig. 1,  $Q^2 = -q^2$  and  $W^2 = (p + q)^2$  are standard DIS variables). The ground state vector mesons,  $V = \rho^0, \omega^0, \varphi^0, J/\Psi, \Upsilon$  are usually supposed to be the  $S$ -wave spin-triplet  $q\bar{q}$  states. However, all the previous theoretical calculations used the  $V\bar{q}q$  vertex  $\phi_V V_\mu \bar{q} \Gamma_\mu q$  with the simplest choice  $\Gamma_\mu = \gamma_\mu$ , which corresponds to a certain mixture of the  $S$ - and  $D$ -wave states, and any discussion of the impact of the  $D$ -wave admixture in the literature is missing (here  $V_\mu$  is the vector meson polarization vector and  $\phi_V$  is the vertex function related to the vector meson LCWF as specified below.).

We report here a derivation of helicity amplitudes for diffractive production of pure  $S$  and  $D$ -wave  $q\bar{q}$  systems for small to moderate momentum transfer  $\Delta$  within the diffraction cone. Understanding production of  $D$ -wave states is a topical issue for several reasons. First, the  $D$ -wave admixture may affect predictions for the ratio  $R = \sigma_L/\sigma_T$ , in which there is a persistent departures of theory from the experiment. To this end we recall that the nonperturbative long-range pion-exchange between light quarks and antiquarks [7] is a natural source of  $S$ - $D$

mixing in the ground state  $\rho^0$  and  $\omega^0$  mesons. Second, different spin properties of  $S$ - and  $D$ -wave production may facilitate as yet unresolved  $D$ -wave vs.  $2S$ -wave assignment of the  $\rho'(1480)$  and  $\rho'(1700)$  and of the  $\omega'(1420)$  and  $\omega'(1600)$  mesons.

In our analysis we rely heavily upon the derivation [8] of amplitudes of the  $s$ -channel helicity conserving (SCHC) and non-conserving (SCHNC) transitions, albeit in slightly different notations. We predict a dramatically different spin dependence for production of the  $S$  and  $D$  wave states, especially the  $Q^2$  dependence of  $R = \sigma_L/\sigma_T$  which derives from the anomalously large higher twist effects in the SCHC amplitude for production of longitudinally polarized vector mesons. Our technique can be readily generalized to higher excited states,  $3^-$  etc, leptonproduction of which is interesting for the fact that they cannot be formed in  $e^+e^-$  annihilation.

A typical leading  $\log\frac{1}{x}$  ( $\text{LL}\frac{1}{x}$ ) pQCD diagram for vector meson production is shown in Fig. 1. We use the standard Sudakov expansion of all the momenta in the two lightcone vectors

$$p' = p - q \frac{p^2}{s}, \quad q' = q + p' \frac{Q^2}{s}$$

such that  $q'^2 = p'^2 = 0$  and  $s = 2p' \cdot q'$ , and the two-dimensional transverse component:  $k = zq' + yp' + k_\perp$ ,  $\kappa = \alpha q' + \beta p' + \kappa_\perp$ ,  $\Delta = \gamma p' + \delta q' + \Delta_\perp$  (with the exception of  $\mathbf{r}$  which is a 3-dimensional vector, see below, hereafter  $\mathbf{k}, \Delta, ..$  always stand for 2-dimensional  $k_\perp, \Delta_\perp$  etc.). The diffractive helicity amplitudes take the form

$$A_{\lambda_V \lambda_\gamma}^{S,D}(x, Q^2, \Delta) = i s \frac{C_F N_c c_V \sqrt{4\pi\alpha_{em}}}{2\pi^2} \int_0^1 \frac{dz}{z(1-z)} \int d^2\mathbf{k} \psi_{S,D}(z, \mathbf{k}) \int \frac{d^2\kappa}{\kappa^4} \alpha_S(\max\{\kappa^2, \mathbf{k}^2 + \overline{Q}^2\}) I_{\lambda_V \lambda_\gamma}^{S,D}(\gamma^* \rightarrow V) \left(1 + i \frac{\pi}{2} \frac{\partial}{\partial \log x}\right) \mathcal{F}(x, \kappa, \Delta), \quad (1)$$

where  $\lambda_V, \lambda_\gamma$  stand for helicities,  $m$  is the quark mass,  $C_F = \frac{N_c^2-1}{2N_c}$  is the Casimir operator,  $N_c = 3$  is the number of colors,  $c_V = \frac{1}{\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{1}{3}, \frac{2}{3}$  for the  $\rho^0, \omega^0, \phi^0, J/\Psi$  mesons,  $\alpha_{em}$  is the fine structure constant,  $\alpha_S$  is the strong coupling and  $\overline{Q}^2 = m^2 + z(1-z)Q^2$  is the relevant hard scale. To the  $\text{LL}\frac{1}{x}$  the lower blob is related to the unintegrated gluon density matrix  $\mathcal{F}(x, \kappa, \Delta)$  [5, 10, 11]. For small  $\Delta$  within the diffraction cone

$$\mathcal{F}(x, \kappa, \Delta) = \frac{\partial G(x, \kappa^2)}{\partial \log \kappa^2} \exp(-\frac{1}{2} B_{3\mathbf{P}} \Delta^2). \quad (2)$$

where  $\partial G/\partial \log \kappa^2$  is the conventional unintegrated gluon structure function and, modulo to a slow Regge growth, the diffraction cone  $B_{3\mathbf{P}} \sim 6 \text{ GeV}^{-2}$  [5].

In the light-cone formalism [9], one first computes the production of an on-mass shell  $q\bar{q}$  pair of invariant mass  $M$  and total momentum  $q_M$ . This amplitude is projected onto the state  $(q\bar{q})_J$  of total angular momentum  $J = 1$  using the running longitudinal and the usual transverse polarization vectors

$$V_L = \frac{1}{M} \left( q' + \frac{\Delta^2 - M^2}{s} p' + \Delta_\perp \right), \quad V_T = V_\perp + \frac{2(\mathbf{V}_\perp \cdot \Delta)}{s} (p' - q'), \quad (3)$$

such that  $(V_T V_L) = (V_T q_M) = (V_L q_M) = 0$ . Then the resulting upper blob  $I(\gamma^* \rightarrow V)$  is contracted with the radial LCWF of the  $q\bar{q}$  Fock state of the vector meson,

$$\psi_{S,D}(z, \mathbf{k}) = \psi_{S,D}(\mathbf{r}^2) = \frac{\phi_{S,D}(\mathbf{r}^2)}{M^2 - m_V^2}. \quad (4)$$

Here  $r = \frac{1}{2}(k_2 - k_1)$ , which in the rest frame is the relative 3-momentum in the  $q\bar{q}$  pair,  $r = (0, \mathbf{r}) = (0, \mathbf{k}, k_z)$ ,  $r^2 = -\mathbf{r}^2$ , and

$$M^2 = 4(m^2 + \mathbf{r}^2) = \frac{m^2 + \mathbf{k}^2}{z(1-z)}.$$

To conform to this procedure, all the occurrences of the vector meson mass  $m_V$  in  $I_{\lambda_V \lambda_\gamma}$  of ref. [8] must be replaced by  $M$ .

A useful normalization of the radial LCWF's  $\psi_{S,D}(\mathbf{r}^2)$  is provided by the  $V \rightarrow e^+e^-$  decay constant,  $\langle 0 | J_{mu}^{em} | V \rangle = f_{c_V} \sqrt{4\pi\alpha_{em}} V_\mu$ :

$$f_S = \frac{N_c}{(2\pi)^3} \int d^3\mathbf{r} \frac{8}{3} (M + m) \psi_S(\mathbf{r}^2), \quad f_D = \frac{N_c}{(2\pi)^3} \int d^3\mathbf{r} \frac{32}{3} \frac{\mathbf{r}^4}{M + 2m} \psi_D(\mathbf{r}^2). \quad (5)$$

The nice observation is that we need not go again through all the calculations of helicity amplitudes. Indeed, the spinor vertices  $\Gamma_\mu^{S,D}$  for the pure  $S$  and  $D$  wave states can be readily obtained from the simplest  $\Gamma_\mu = \gamma_\mu$  used in [8]. Following [9], it can be easily shown that

$$\Gamma_\mu^S = \gamma_\mu - \frac{2r_\mu}{M + 2m} = \mathcal{S}_{\mu\nu} \gamma_\nu; \quad \mathcal{S}_{\mu\nu} = g_{\mu\nu} - \frac{2r_\mu r_\nu}{m(M + 2m)}. \quad (6)$$

Here we made use of  $r^\mu \gamma_\mu = m$  and  $(q_M \cdot r) = 0$ . Once the  $S$ -wave is constructed, the spinor structure for a  $D$ -wave state can be readily obtained by contracting the  $S$ -wave vertex with  $3r_\mu r_\nu + g_{\mu\nu} \mathbf{r}^2$  with the result

$$\Gamma_\mu^D = \mathbf{r}^2 \gamma_\mu + (M + m) r_\mu = \mathcal{D}_{\mu\nu} \gamma_\nu; \quad \mathcal{D}_{\mu\nu} = \mathbf{r}^2 g_{\mu\nu} + \frac{M + m}{m} r_\mu r_\nu. \quad (7)$$

Consequently, the answers for either  $S$  or  $D$ -wave production amplitudes can be immediately obtained from the expressions given in [8] by substitutions  $V_\mu^* \rightarrow V_\nu^* \mathcal{S}_{\nu\mu}$ ,  $V_\mu^* \rightarrow V_\nu^* \mathcal{D}_{\nu\mu}$  for  $S$  and  $D$ -wave states respectively.

In terms of diffractive amplitudes  $\Phi_1$  and  $\Phi_2$  defined in [8], we find for  $S$ -wave vector mesons (here  $T$  stands for the transverse polarization)

$$\begin{aligned} I_{0L}^S &= -4QMz^2(1-z)^2\Phi_2 \left[ 1 + \frac{(1-2z)^2m}{2z(1-z)(M+2m)} \right], \\ I_{TT}^S &= \left\{ (\mathbf{V}^* \mathbf{e}) [m^2\Phi_2 + (\mathbf{k}\Phi_1)] + (1-2z)^2(\mathbf{V}^* \mathbf{k})(\mathbf{e}\Phi_1) \frac{M}{M+2m} \right. \\ &\quad \left. - (\mathbf{e}\mathbf{k})(\mathbf{V}^* \Phi_1) + \frac{2m}{M+2m} (\mathbf{V}^* \mathbf{k})(\mathbf{e}\mathbf{k})\Phi_2 \right\}, \\ I_{0T}^S &= -2z(1-z)(2z-1)M(\mathbf{e}\Phi_1) \left[ 1 + \frac{(1-2z)^2m}{2z(1-z)(M+2m)} \right] + \frac{Mm}{M+2m} (2z-1)(\mathbf{e}\mathbf{k})\Phi_2, \\ I_{TL}^S &= 2Qz(1-z)(2z-1)(\mathbf{V}^* \mathbf{k})\Phi_2 \frac{M}{M+2m}. \quad (8) \end{aligned}$$

Because the difference between  $\Gamma_\mu^S$  and  $\gamma_\mu$  is a relativistic correction, the results for the  $S$ -wave vector mesons differ from those found in [8] only by a small relativistic corrections  $\propto \mathbf{r}^2/M^2$ . The exceptional case is suppression of  $I_{TL}^S$  by factor  $M/(2m + M) \sim 0.5$ .

We skip the twist expansion for  $S$ -wave amplitudes, which can easily be done following [8], and proceed to the much more interesting case of  $D$ -wave mesons, for which

$$\begin{aligned}
I_{0L}^D &= -QMz(1-z) \left( \mathbf{k}^2 - \frac{4m}{M}k_z^2 \right) \Phi_2, \\
I_{TT}^D &= \left\{ (\mathbf{V}^* \mathbf{e}) \mathbf{r}^2 [m^2 \Phi_2 + (\mathbf{k} \Phi_1)] + (1-2z)^2 (\mathbf{r}^2 + m^2 + Mm) (\mathbf{V}^* \mathbf{k}) (\mathbf{e} \Phi_1) \right. \\
&\quad \left. - \mathbf{r}^2 (\mathbf{e} \mathbf{k}) (\mathbf{V}^* \Phi_1) - m(M+m) (\mathbf{V}^* \mathbf{k}) (\mathbf{e} \mathbf{k}) \Phi_2 \right\}, \\
I_{0T}^D &= -\frac{2z-1}{2} M \left\{ (\mathbf{e} \Phi_1) (\mathbf{k}^2 - \frac{4m}{M}k_z^2) + m(M+m) (\mathbf{e} \mathbf{k}) \Phi_2 \right\}, \\
I_{TL}^D &= 2Qz(1-z)(2z-1) (\mathbf{V}^* \mathbf{k}) (\mathbf{r}^2 + m^2 + Mm) \Phi_2, \tag{9}
\end{aligned}$$

The novel features of these amplitudes are best seen in the twist expansion in inverse powers of the hard scale  $\overline{Q}^2$ . As it was noted in [8], in all cases but the double helicity flip the dominant twist amplitudes come from the leading  $\log \overline{Q}^2$  (LL $\overline{Q}^2$ ) region of  $\mathbf{k}^2 \sim R_V^{-2}$ ,  $\Delta^2 \ll \kappa^2 \ll \overline{Q}^2$ . The closer inspection of our  $I_{\lambda_V \lambda_\gamma}^D$  shows that the seemingly leading interference with the dominant  $S$ -wave component in the photon always appears in the quadrupole combination  $2k_z^2 - \mathbf{k}^2$ . Since the integration over quark loop can be cast in form  $d^3 \mathbf{r}$ , such quadrupole combinations vanish after angular integration. As a result, the abnormally large higher twist contributions  $\propto M^2/(M^2 + Q^2)$  with large numerical factors come into play and significantly modify the  $Q^2$  dependence of amplitudes for production of longitudinally polarized vector mesons:

$$I_{0L}^D = -\frac{Q}{M} \cdot \frac{32\mathbf{r}^4}{15(M^2 + Q^2)^2} \cdot \left( 1 - 8 \frac{M^2}{M^2 + Q^2} \right) \kappa^2, \tag{10}$$

$$I_{\pm\pm}^D = (\mathbf{V}^* \mathbf{e}) \cdot \frac{32\mathbf{r}^4}{15(M^2 + Q^2)^2} \cdot \left( 15 + 4 \frac{M^2}{M^2 + Q^2} \right) \kappa^2, \tag{11}$$

$$I_{\pm L}^D = -\frac{32\mathbf{r}^4}{15(M^2 + Q^2)^2} \cdot \frac{24Q(\mathbf{V}^* \Delta)}{M^2 + Q^2} \kappa^2, \tag{12}$$

$$I_{L\pm}^D = \frac{32\mathbf{r}^4}{15(M^2 + Q^2)^2} \cdot \frac{8(\mathbf{e} \Delta)}{M} \left( 1 + 3 \frac{M^2}{M^2 + Q^2} \right) \kappa^2, \tag{13}$$

$$I_{\pm\mp}^D = (\mathbf{V}^* \Delta)(\mathbf{e} \Delta) \cdot \frac{32\mathbf{r}^4}{15(M^2 + Q^2)^2} \cdot \left( 1 - \frac{96}{7} \frac{\kappa^2 \mathbf{r}^2}{M^2(M^2 + Q^2)} \right). \tag{14}$$

In a close similarity to the  $S$ -wave case [8], the leading twist double-helicity flip amplitude is dominated by soft gluon exchange, the LL $\overline{Q}^2$  component is of higher twist.

In order to emphasize striking difference between the  $D$ -wave and  $S$ -wave state amplitudes, we focus on nonrelativistic heavy quarkonia, where  $M^2 \approx m_V^2$ , although all the qualitative results hold for light vector mesons too. For the illustration purposes, we evaluated the ratios of helicity amplitudes,  $\rho_{D/S} = f_S A^D / f_D A^S$ , for the the harmonic oscillator wave functions:

$$\begin{aligned}
\rho_{0L}(D/S) &= \frac{1}{5} \left( 1 - 8 \frac{m_V^2}{Q^2 + m_V^2} \right), \\
\rho_{\pm\pm}(D/S) &= 3 \left( 1 + \frac{4}{15} \frac{m_V^2}{Q^2 + m_V^2} \right),
\end{aligned}$$

$$\begin{aligned}\rho_{0\pm}(D/S) &= -\frac{1}{5}(m_V a_S)^2 \left(1 + 3 \frac{m_V^2}{Q^2 + m_V^2}\right), \\ \rho_{\pm L}(D/S) &= \frac{3}{40}(m_V a_S)^4.\end{aligned}\tag{15}$$

First,  $A_{0L}$  changes the sign at  $Q^2 \sim 7m_V^2$ . The ratio  $R^D = \sigma_L/\sigma_T$  has thus a non-monotonous  $Q^2$  behavior and  $R^D \ll R^S$ . Furthermore,  $R^D \lesssim 1$  in a broad range of  $Q^2 \lesssim 225m_V^2$ . Whereas in heavy quarkonia the  $S$ - $D$  mixing is arguably weak [12], in light  $\rho^0, \omega^0$  even a relatively weak  $S$ - $D$  mixing could have a substantial impact on  $R$ . Second, all the  $D$ -wave amplitudes, SCHC and SCHNC alike, with exception of the higher twist component of double-helicity flip, are proportional to  $\mathbf{r}^4$  and, in view of eq. (5), to the decay constant  $f_D$ . In contrast to that, in the  $S$ -wave case the spin-flip amplitudes for heavy quarkonia are suppressed by nonrelativistic Fermi motion [8]. The relevant suppression parameter is  $\sim 1/(a_S m_V)^2$ , where  $a_S$  is the radius of the  $1S$  state. For this reason, for  $D$ -wave states the SCHNC effects are much stronger. For instance, for the charmonium  $(m_V a_S)^2 \approx 27$ , see [12].

To summarize, we found dramatically different spin properties of diffractive leptonproduction of the  $S$  and  $D$  wave states of vector mesons. We predict very small  $R^D = \sigma_L/\sigma_T$  and very strong breaking of  $s$ -channel helicity conservation in production of  $D$ -wave states. Higher twist effects in production of longitudinally polarized  $D$ -wave vector mesons are found to be abnormally large. Consequently, the distinct spin properties of  $D$ -wave vector mesons in diffractive DIS offer an interesting way to discern  $S$  and  $D$ -wave meson states, which are indistinguishable at  $e^+e^-$  colliders.

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**Figure caption:**

Fig.1: One of the four Feynman diagrams for the vector meson production  $\gamma^* p \rightarrow V p'$  via QCD two-gluon pomeron exchange.